CH 3: Steady Heat Conduction

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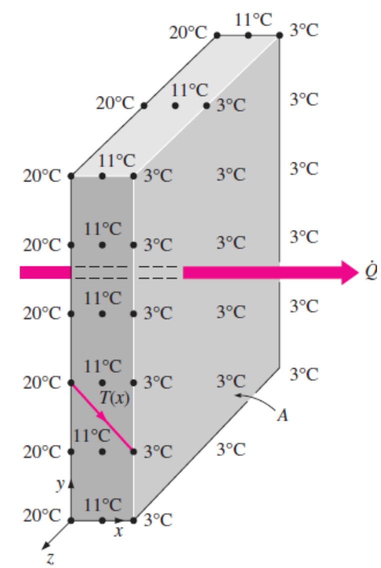
- Heat Trnasfer problems can be modelled as an electric circuit models
- The thermal resistance corresponds to electrical resistance
- Temperature difference corresponds to voltage
- The heat transfer rate corresponds to electric current

Steady Heat Conduction in Plane Walls

- We intuitively feel that heat transfer through the wall is in the *normal direction* to the wall surface, and no significant heat transfer takes place in the wall in other directions.
- Recall: heat transfer in a certain direction is driven by the *temperature gradient in that direction.*



- The energy balance for the wall can be expressed as:

$$\begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{into the wall}\end{pmatrix} - \begin{pmatrix}\text{Rate of}\\ \text{heat transfer}\\ \text{out of the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of change}\\ \text{of the energy}\\ \text{of the wall}\end{pmatrix}$$

or

$$\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}} = \frac{dE_{\text{wall}}}{dt}$$

But $dE_{\text{wall}}/dt = 0$ for *steady* operation

- Which means in=out and we get

$$\dot{Q}_{\text{cond, wall}} = -kA\,\frac{dT}{dx} \qquad \text{(W)}$$

Separating the variables in the above equation and integrating from $x = 0$, where $T(0) = T_1$, to $x = L$, where $T(L) = T_2$, we get

$$\int_{x=0}^{L} \dot{Q}_{\text{cond, wall}}\, dx = -\int_{T=T_1}^{T_2} kA\, dT$$

Performing the integrations and rearranging gives

$$\dot{Q}_{\text{cond, wall}} = kA\frac{T_1 - T_2}{L} \qquad \text{(W)} \tag{3-3}$$

The Thermal Resistance Concept

Equation 3–3 for heat conduction through a plane wall can be rearranged as

$$\dot{Q}_{\text{cond, wall}} = \frac{T_1 - T_2}{R_{\text{wall}}} \qquad \text{(W)} \tag{3-4}$$

where

$$R_{\text{wall}} = \frac{L}{kA} \qquad (°\text{C/W}) \tag{3-5}$$

is the *thermal resistance* of the wall against heat conduction or simply the **conduction resistance** of the wall. Note that the thermal resistance of a medium depends on the *geometry* and the *thermal properties* of the medium.

The equation above for heat flow is analogous to the relation for *electric current flow I*, expressed as

$$I = \frac{\mathbf{V}_1 - \mathbf{V}_2}{R_e} \tag{3-6}$$

where $R_e = L/\sigma_e A$ is the *electric resistance* and $\mathbf{V}_1 - \mathbf{V}_2$ is the *voltage difference* across the resistance (σ_e is the electrical conductivity). Thus, the *rate of heat transfer* through a layer corresponds to the *electric current*, the *thermal resistance* corresponds to *electrical resistance*, and the *temperature difference* corresponds to *voltage difference* across the layer (Fig. 3–3).

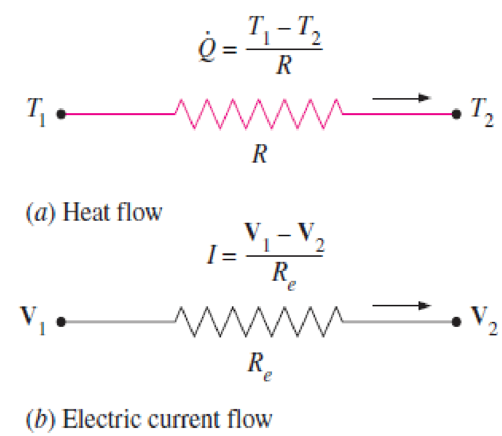


FIGURE 3–3
Analogy between thermal and electrical resistance concepts.

- For convection heat transfer Newton's law of cooling can be re-arranged as:

$$\dot{Q}_{conv} = \frac{T_s - T_\infty}{R_{conv}} \qquad (W) \qquad \text{(3-7)}$$

where

$$R_{conv} = \frac{1}{hA_s} \qquad (°C/W) \qquad \text{(3-8)}$$

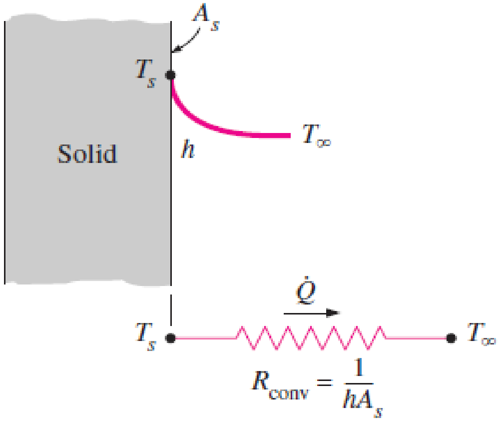


FIGURE 3–4
Schematic for convection resistance at a surface.

is the *thermal resistance* of the surface against heat convection, or simply the **convection resistance** of the surface (Fig. 3–4). Note that when the convection heat transfer coefficient is very large ($h \to \infty$), the convection resistance becomes *zero* and $T_s \approx T_\infty$. That is, the surface offers *no resistance to convection*, and thus it does not slow down the heat transfer process. This situation is approached in practice at surfaces where boiling and condensation occur. Also note that the surface does not have to be a plane surface. Equation 3–8 for convection resistance is valid for surfaces of any shape, provided that the assumption of h = constant and uniform is reasonable.

- For radiation heat transfer we get

$$\dot{Q}_{rad} = \varepsilon\sigma A_s (T_s^4 - T_{surr}^4) = h_{rad} A_s (T_s - T_{surr}) = \frac{T_s - T_{surr}}{R_{rad}} \qquad (W) \qquad \textbf{(3-9)}$$

where

$$R_{rad} = \frac{1}{h_{rad} A_s} \qquad (K/W) \qquad \textbf{(3-10)}$$

is the *thermal resistance* of a surface against radiation, or the *radiation resistance*, and

$$h_{rad} = \frac{\dot{Q}_{rad}}{A_s(T_s - T_{surr})} = \varepsilon\sigma(T_s^2 + T_{surr}^2)(T_s + T_{surr}) \qquad (W/m^2 \cdot K) \qquad \textbf{(3-11)}$$

is the **radiation heat transfer coefficient.** Note that both T_s and T_{surr} *must* be in K in the evaluation of h_{rad}. The definition of the radiation heat transfer coefficient enables us to express radiation conveniently in an analogous manner to convection in terms of a temperature difference. But h_{rad} depends strongly on temperature while h_{conv} usually does not.

- Combined heat transfer can be re-written as

A surface exposed to the surrounding air involves convection and radiation simultaneously, and the total heat transfer at the surface is determined by adding (or subtracting, if in the opposite direction) the radiation and convection components. The convection and radiation resistances are parallel to each other, as shown in Fig. 3–5, and may cause some complication in the thermal resistance network. When $T_{surr} \approx T_\infty$, the radiation effect can properly be accounted for by replacing h in the convection resistance relation by

$$h_{combined} = h_{conv} + h_{rad} \qquad (W/m^2 \cdot K) \tag{3-12}$$

where $h_{combined}$ is the **combined heat transfer coefficient.** This way all the complications associated with radiation are avoided.

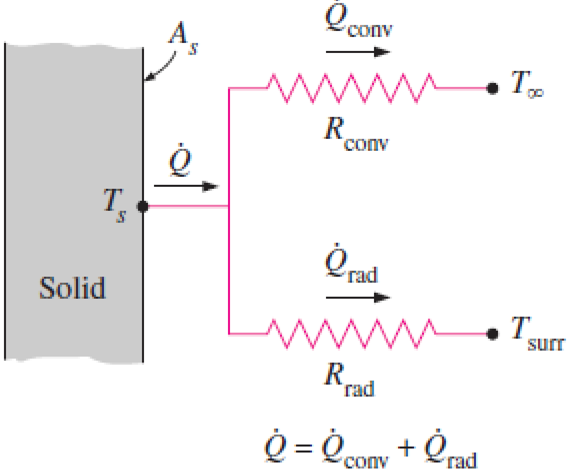


FIGURE 3–5
Schematic for convection and radiation resistances at a surface.

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{conv,1} + R_{wall} + R_{conv,2}}$$

$$I = \frac{\mathcal{V}_1 - \mathcal{V}_2}{R_{e,1} + R_{e,2} + R_{e,3}}$$

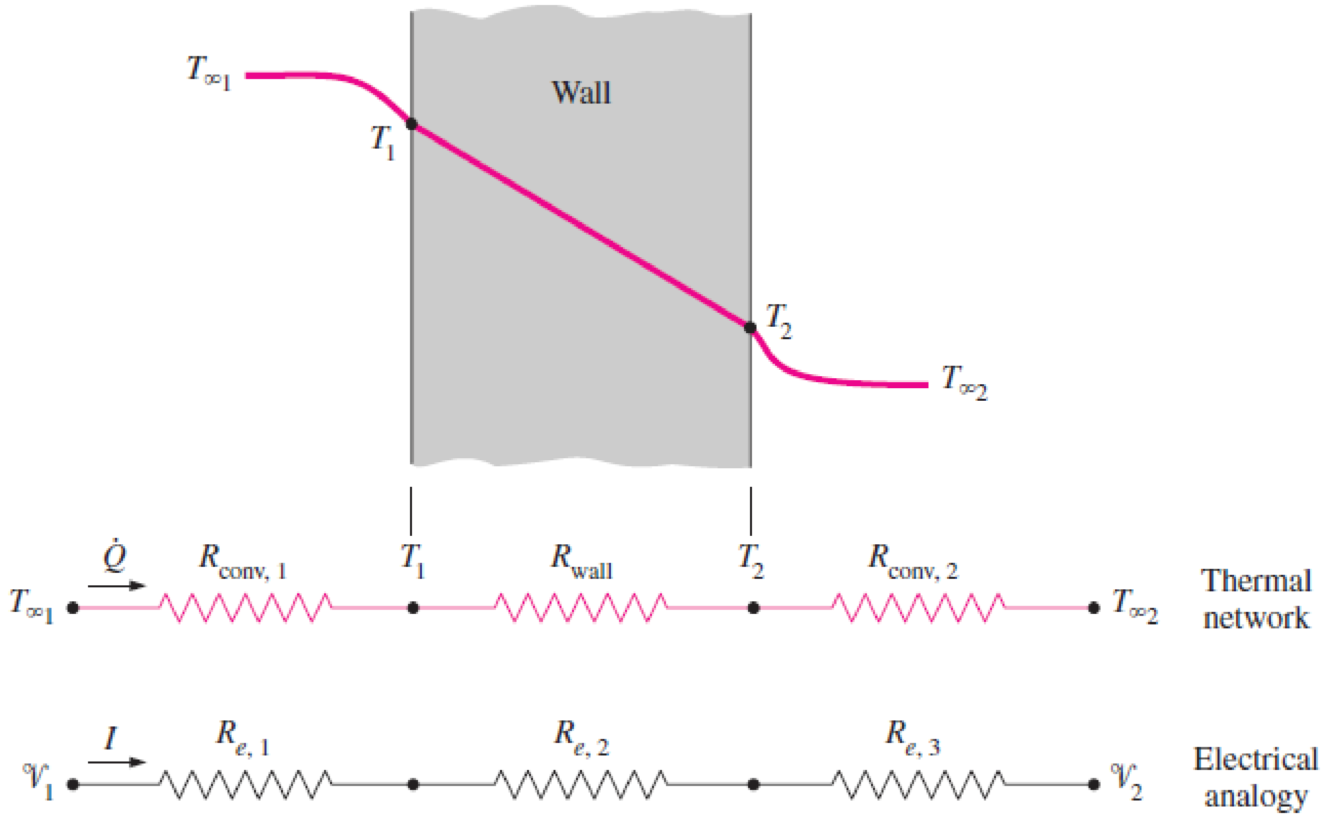


FIGURE 3–6

The thermal resistance network for heat transfer through a plane wall subjected to convection on both sides, and the electrical analogy.

Thermal Resistance Network

Under steady conditions we have

$$\begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{into the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat conduction}\\ \text{through the wall}\end{pmatrix} = \begin{pmatrix}\text{Rate of}\\ \textit{heat convection}\\ \text{from the wall}\end{pmatrix}$$

or

$$\dot{Q} = h_1 A(T_{\infty 1} - T_1) = kA\frac{T_1 - T_2}{L} = h_2 A(T_2 - T_{\infty 2}) \qquad \textbf{(3-13)}$$

which can be rearranged as

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{1/h_1 A} = \frac{T_1 - T_2}{L/kA} = \frac{T_2 - T_{\infty 2}}{1/h_2 A}$$

$$= \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} = \frac{T_1 - T_2}{R_{\text{wall}}} = \frac{T_2 - T_{\infty 2}}{R_{\text{conv, 2}}} \qquad \textbf{(3-14)}$$

Adding the numerators and denominators yields (Fig. 3–7)

$$\dot{Q} = \frac{T_{\infty} - T_{\infty 2}}{R_{\text{total}}} \qquad \text{(W)} \qquad \textbf{(3-15)}$$

If

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n} = c$$

then

$$\frac{a_1 + a_2 + \cdots + a_n}{b_1 + b_2 + \cdots + b_n} = c$$

For example,

$$\frac{1}{4} = \frac{2}{8} = \frac{5}{20} = 0.25$$

and

$$\frac{1 + 2 + 5}{4 + 8 + 20} = 0.25$$

FIGURE 3–7
A useful mathematical identity.

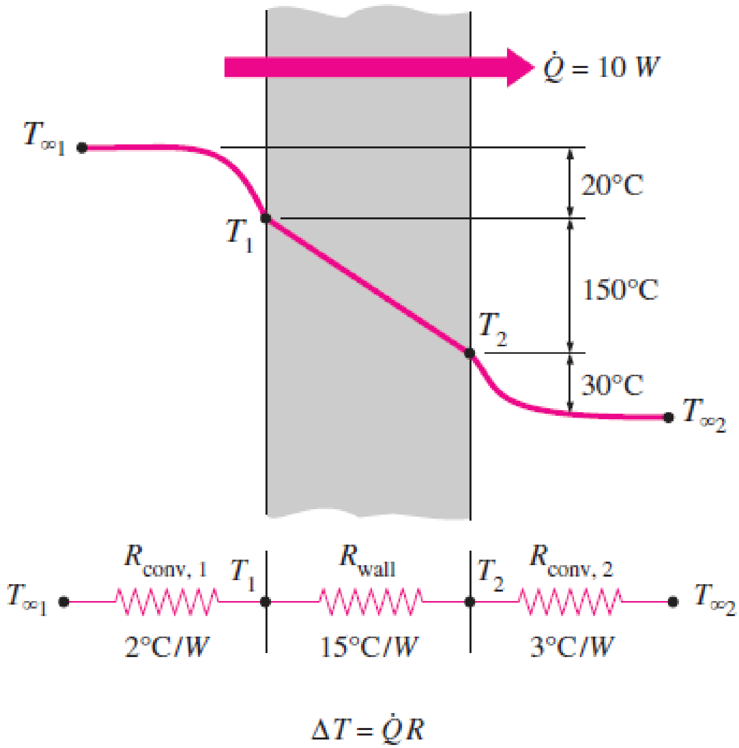


FIGURE 3–8

The temperature drop across a layer is proportional to its thermal resistance.

where

$$R_{total} = R_{conv,\,1} + R_{wall} + R_{conv,\,2} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \qquad (°C/W) \qquad \textbf{(3-16)}$$

- Equation 3-15 can be re-arranged as:

$$\Delta T = \dot{Q} R \qquad (°C) \qquad (3\text{-}17)$$

which indicates that the *temperature drop* across any layer is equal to the *rate of heat transfer* times the *thermal resistance* across that layer (Fig. 3–8). You may recall that this is also true for voltage drop across an electrical resistance when the electric current is constant.

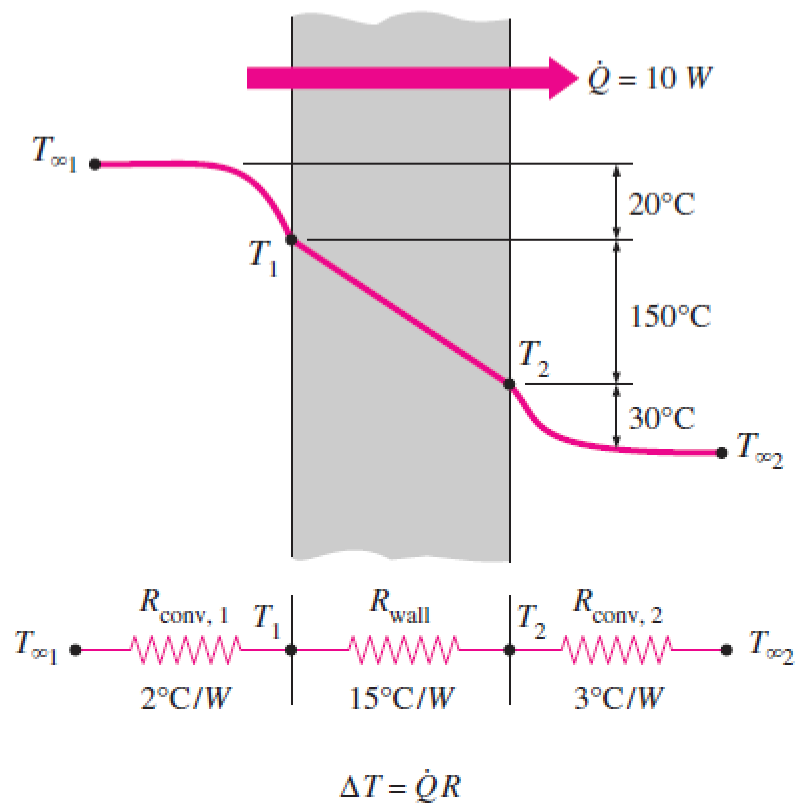


FIGURE 3–8
The temperature drop across a layer is proportional to its thermal resistance.

It is sometimes convenient to express heat transfer through a medium in an analogous manner to Newton's law of cooling as

$$\dot{Q} = UA\,\Delta T \qquad \text{(W)} \tag{3-18}$$

where U is the **overall heat transfer coefficient.** A comparison of Eqs. 3–15 and 3–18 reveals that

$$UA = \frac{1}{R_{\text{total}}} \tag{3-19}$$

Multi Plane Walls

Consider a plane wall that consists of two layers (such as a brick wall with a layer of insulation). The rate of steady heat transfer through this two-layer composite wall can be expressed as (Fig. 3–9)

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} \qquad \textbf{(3-21)}$$

where R_{total} is the *total thermal resistance,* expressed as

$$R_{total} = R_{conv,\,1} + R_{wall,\,1} + R_{wall,\,2} + R_{conv,\,2}$$
$$= \frac{1}{h_1 A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{1}{h_2 A} \qquad \textbf{(3-22)}$$

The subscripts 1 and 2 in the R_{wall} relations above indicate the first and the second layers, respectively. We could also obtain this result by following the approach used above for the single-layer case by noting that the rate of steady heat transfer $\dot{Q}$ through a multilayer medium is constant, and thus it must be the same through each layer. Note from the thermal resistance network that the resistances are *in series,* and thus the *total thermal resistance* is simply the *arithmetic sum* of the individual thermal resistances in the path of heat flow.

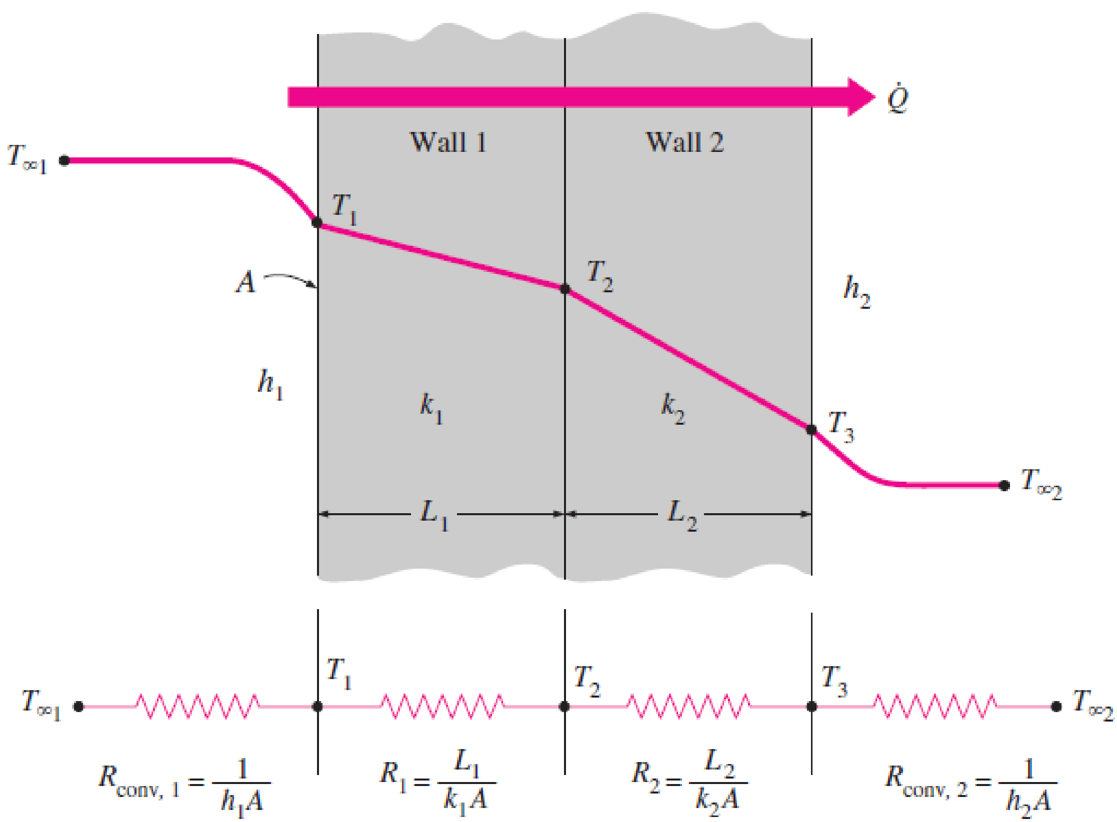


FIGURE 3–9
The thermal resistance network for heat transfer through a two-layer plane wall subjected to convection on both sides.

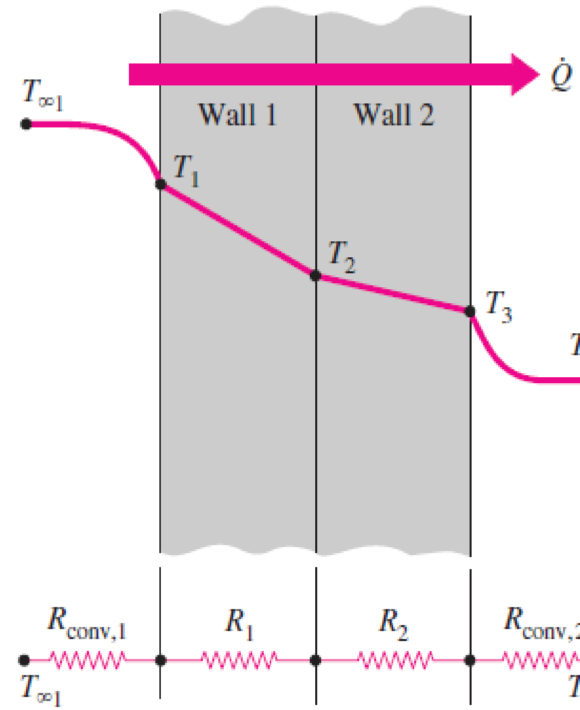
Once $\dot{Q}$ is *known,* an unknown surface temperature T_j at any surface or interface j can be determined from

$$\dot{Q} = \frac{T_i - T_j}{R_{\text{total},\, i-j}} \tag{3-23}$$

where T_i is a *known* temperature at location i and $R_{\text{total},\, i-j}$ is the total thermal resistance between locations i and j. For example, when the fluid temperatures $T_{\infty 1}$ and $T_{\infty 2}$ for the two-layer case shown in Fig. 3–9 are available and $\dot{Q}$ is calculated from Eq. 3–21, the interface temperature T_2 between the two walls can be determined from (Fig. 3–10)

$$\dot{Q} = \frac{T_{\infty 1} - T_2}{R_{\text{conv},\,1} + R_{\text{wall},\,1}} = \frac{T_{\infty 1} - T_2}{\dfrac{1}{h_1 A} + \dfrac{L_1}{k_1 A}} \tag{3-24}$$

The temperature drop across a layer is easily determined from Eq. 3–17 by multiplying $\dot{Q}$ by the thermal resistance of that layer.



To find T_1: $\dot{Q} = \dfrac{T_{\infty 1} - T_1}{R_{\text{conv},1}}$

To find T_2: $\dot{Q} = \dfrac{T_{\infty 1} - T_2}{R_{\text{conv},1} + R_1}$

To find T_3: $\dot{Q} = \dfrac{T_3 - T_{\infty 2}}{R_{\text{conv},2}}$

FIGURE 3–10
The evaluation of the surface and interface temperatures when $T_{\infty 1}$ and $T_{\infty 2}$ are given and $\dot{Q}$ is calculated.

EXAMPLE 3–1 Heat Loss through a Wall

Consider a 3-m-high, 5-m-wide, and 0.3-m-thick wall whose thermal conductivity is $k = 0.9$ W/m · °C (Fig. 3–11). On a certain day, the temperatures of the inner and the outer surfaces of the wall are measured to be 16°C and 2°C, respectively. Determine the rate of heat loss through the wall on that day.

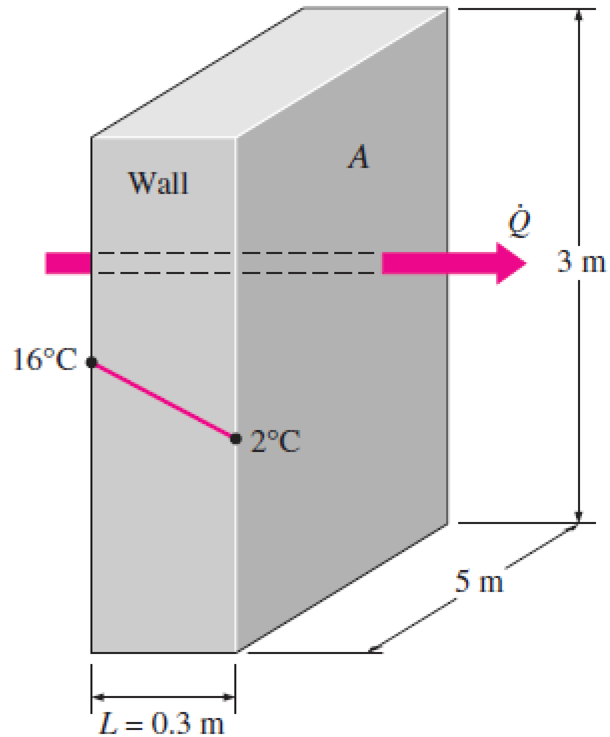


FIGURE 3–11
Schematic for Example 3–1.

SOLUTION The two surfaces of a wall are maintained at specified temperatures. The rate of heat loss through the wall is to be determined.

Assumptions **1** Heat transfer through the wall is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.9$ W/m · °C.

Analysis Noting that the heat transfer through the wall is by conduction and the area of the wall is $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$, the steady rate of heat transfer through the wall can be determined from Eq. 3–3 to be

$$\dot{Q} = kA\frac{T_1 - T_2}{L} = (0.9 \text{ W/m} \cdot {}^\circ\text{C})(15 \text{ m}^2)\frac{(16 - 2)^\circ\text{C}}{0.3 \text{ m}} = \mathbf{630\ W}$$

We could also determine the steady rate of heat transfer through the wall by making use of the thermal resistance concept from

$$\dot{Q} = \frac{\Delta T_{\text{wall}}}{R_{\text{wall}}}$$

where

$$R_{\text{wall}} = \frac{L}{kA} = \frac{0.3 \text{ m}}{(0.9 \text{ W/m} \cdot {}^\circ\text{C})(15 \text{ m}^2)} = 0.02222^\circ\text{C/W}$$

Substituting, we get

$$\dot{Q} = \frac{(16 - 2)^\circ\text{C}}{0.02222^\circ\text{C/W}} = 630 \text{ W}$$

EXAMPLE 3–2 Heat Loss through a Single-Pane Window

Consider a 0.8-m-high and 1.5-m-wide glass window with a thickness of 8 mm and a thermal conductivity of $k = 0.78$ W/m · °C. Determine the steady rate of heat transfer through this glass window and the temperature of its inner surface for a day during which the room is maintained at 20°C while the temperature of the outdoors is −10°C. Take the heat transfer coefficients on the inner and outer surfaces of the window to be $h_1 = 10$ W/m^2 · °C and $h_2 = 40$ W/m^2 · °C, which includes the effects of radiation.

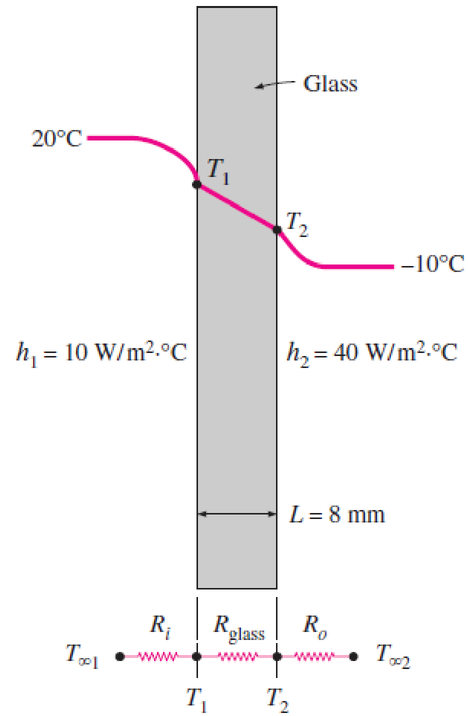


FIGURE 3–12
Schematic for Example 3–2.

Assumptions **1** Heat transfer through the window is steady since the surface temperatures remain constant at the specified values. **2** Heat transfer through the wall is one-dimensional since any significant temperature gradients will exist in the direction from the indoors to the outdoors. **3** Thermal conductivity is constant.

Properties The thermal conductivity is given to be $k = 0.78$ W/m · °C.

Analysis This problem involves conduction through the glass window and convection at its surfaces, and can best be handled by making use of the thermal resistance concept and drawing the thermal resistance network, as shown in Fig. 3–12. Noting that the area of the window is $A = 0.8 \text{ m} \times 1.5 \text{ m} = 1.2 \text{ m}^2$, the individual resistances are evaluated from their definitions to be

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.08333^\circ\text{C/W}$$

$$R_{\text{glass}} = \frac{L}{kA} = \frac{0.008 \text{ m}}{(0.78 \text{ W/m} \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.00855^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(40 \text{ W/m}^2 \cdot {}^\circ\text{C})(1.2 \text{ m}^2)} = 0.02083^\circ\text{C/W}$$

Noting that all three resistances are in series, the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{glass}} + R_{\text{conv, 2}} = 0.08333 + 0.00855 + 0.02083 \\ &= 0.1127^\circ\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the window becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]^\circ\text{C}}{0.1127^\circ\text{C/W}} = \mathbf{266 \text{ W}}$$

Knowing the rate of heat transfer, the inner surface temperature of the window glass can be determined from

$$\dot{Q} = \frac{T_{\infty 1} - T_1}{R_{\text{conv, 1}}} \longrightarrow \begin{aligned} T_1 &= T_{\infty 1} - \dot{Q} R_{\text{conv, 1}} \\ &= 20^\circ\text{C} - (266 \text{ W})(0.08333^\circ\text{C/W}) \\ &= \mathbf{-2.2^\circ C} \end{aligned}$$

Thermal Contact Resistance

- In the analysis of heat conduction through multilayer solids, we assumed "perfect contact" and thus no temperature drop at the interface.
- In reality, however, even flat surfaces that appear smooth to the eye turn out to be rather rough when examined under a microscope.
- Thus, an interface offers some resistance to heat transfer, and this resistance per unit interface area is called the **thermal contact resistance.**

Consider heat transfer through two metal rods of cross-sectional area A that are pressed against each other. Heat transfer through the interface of these two rods is the sum of the heat transfers through the *solid contact spots* and the *gaps* in the noncontact areas and can be expressed as

$$\dot{Q} = \dot{Q}_{\text{contact}} + \dot{Q}_{\text{gap}} \tag{3-25}$$

It can also be expressed in an analogous manner to Newton's law of cooling as

$$\dot{Q} = h_c A \, \Delta T_{\text{interface}} \tag{3-26}$$

where A is the apparent interface area (which is the same as the cross-sectional area of the rods) and $\Delta T_{\text{interface}}$ is the effective temperature difference at the interface. The quantity h_c, which corresponds to the convection heat transfer coefficient, is called the **thermal contact conductance** and is expressed as

$$h_c = \frac{\dot{Q}/A}{\Delta T_{\text{interface}}} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C}) \tag{3-27}$$

It is related to thermal contact resistance by

$$R_c = \frac{1}{h_c} = \frac{\Delta T_{\text{interface}}}{\dot{Q}/A} \qquad (\text{m}^2 \cdot {}^\circ\text{C/W}) \tag{3-28}$$

TABLE 3–1

Thermal contact conductance for aluminum plates with different fluids at the interface for a surface roughness of 10 μm and interface pressure of 1 atm (from Fried, Ref. 5)

Fluid at the Interface	Contact Conductance, h_c, $W/m^2 \cdot °C$
Air	3640
Helium	9520
Hydrogen	13,900
Silicone oil	19,000
Glycerin	37,700

EXAMPLE 3–4 Equivalent Thickness for Contact Resistance

The thermal contact conductance at the interface of two 1-cm-thick aluminum plates is measured to be 11,000 W/m^2 · °C. Determine the thickness of the aluminum plate whose thermal resistance is equal to the thermal resistance of the interface between the plates (Fig. 3–17).

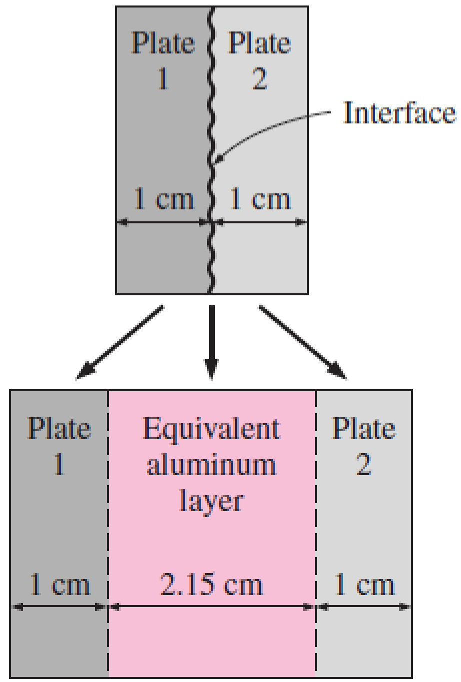


FIGURE 3–17
Schematic for Example 3–4.

SOLUTION The thickness of the aluminum plate whose thermal resistance is equal to the thermal contact resistance is to be determined.

Properties The thermal conductivity of aluminum at room temperature is $k = 237$ W/m · °C (Table A-3).

Analysis Noting that thermal contact resistance is the inverse of thermal contact conductance, the thermal contact resistance is

$$R_c = \frac{1}{h_c} = \frac{1}{11{,}000 \text{ W/m}^2 \cdot {}^\circ\text{C}} = 0.909 \times 10^{-4} \text{ m}^2 \cdot {}^\circ\text{C/W}$$

For a unit surface area, the thermal resistance of a flat plate is defined as

$$R = \frac{L}{k}$$

where L is the thickness of the plate and k is the thermal conductivity. Setting $R = R_c$, the equivalent thickness is determined from the relation above to be

$$L = kR_c = (237 \text{ W/m} \cdot {}^\circ\text{C})(0.909 \times 10^{-4} \text{ m}^2 \cdot {}^\circ\text{C/W}) = 0.0215 \text{ m} = \mathbf{2.15 \text{ cm}}$$

Generalized Thermal Resistance Network

Consider the composite wall shown in Fig. 3–19, which consists of two parallel layers. The thermal resistance network, which consists of two parallel resistances, can be represented as shown in the figure. Noting that the total heat transfer is the sum of the heat transfers through each layer, we have

$$\dot{Q} = \dot{Q}_1 + \dot{Q}_2 = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2} = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \quad \textbf{(3-29)}$$

Utilizing electrical analogy, we get

$$\dot{Q} = \frac{T_1 - T_2}{R_{\text{total}}} \quad \textbf{(3-30)}$$

where

$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} \longrightarrow R_{\text{total}} = \frac{R_1 R_2}{R_1 + R_2} \quad \textbf{(3-31)}$$

since the resistances are in parallel.

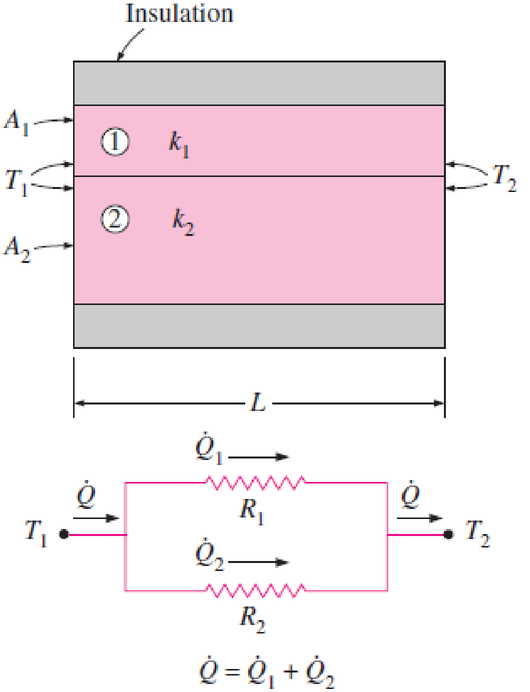


FIGURE 3–19
Thermal resistance network for two parallel layers.

Now consider the combined series-parallel arrangement shown in Fig. 3–20. The total rate of heat transfer through this composite system can again be expressed as

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \tag{3-32}$$

where

$$R_{\text{total}} = R_{12} + R_3 + R_{\text{conv}} = \frac{R_1 R_2}{R_1 + R_2} + R_3 + R_{\text{conv}} \tag{3-33}$$

and

$$R_1 = \frac{L_1}{k_1 A_1}, \qquad R_2 = \frac{L_2}{k_2 A_2}, \qquad R_3 = \frac{L_3}{k_3 A_3}, \qquad R_{\text{conv}} = \frac{1}{hA_3} \tag{3-34}$$

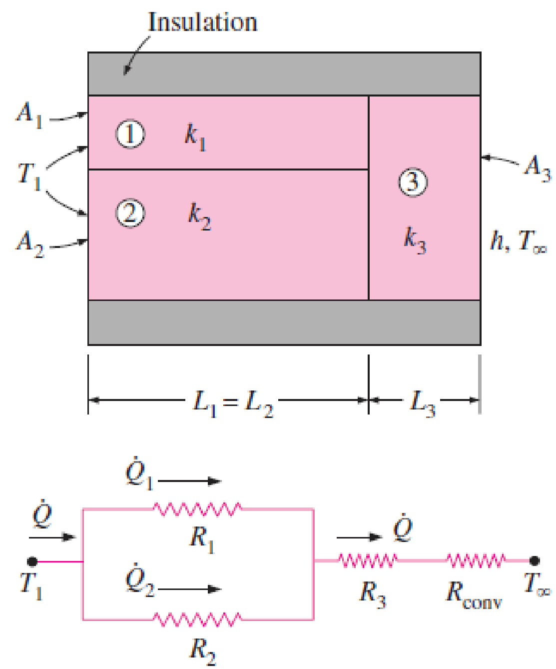


FIGURE 3–20
Thermal resistance network for combined series-parallel arrangement.

EXAMPLE 3–6 Heat Loss through a Composite Wall

A 3-m-high and 5-m-wide wall consists of long 16-cm × 22-cm cross section horizontal bricks (k = 0.72 W/m · °C) separated by 3-cm-thick plaster layers (k = 0.22 W/m · °C). There are also 2-cm-thick plaster layers on each side of the brick and a 3-cm-thick rigid foam (k = 0.026 W/m · °C) on the inner side of the wall, as shown in Fig. 3–21. The indoor and the outdoor temperatures are 20°C and −10°C, and the convection heat transfer coefficients on the inner and the outer sides are h_1 = 10 W/m^2 · °C and h_2 = 25 W/m^2 · °C, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

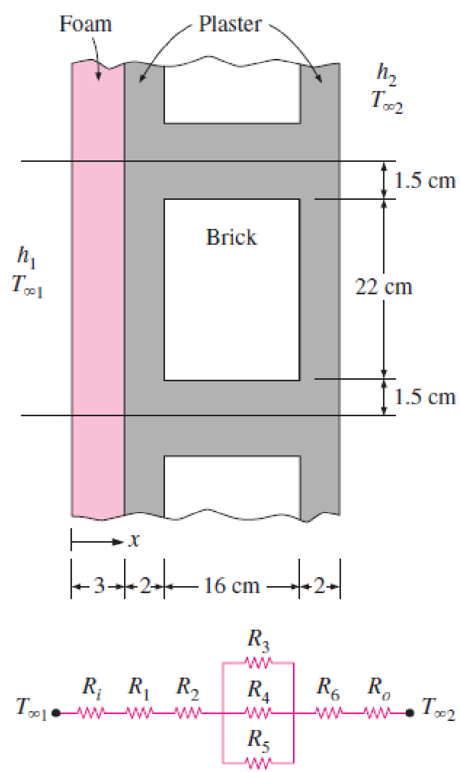


FIGURE 3–21

Schematic for Example 3–6.

SOLUTION The composition of a composite wall is given. The rate of heat transfer through the wall is to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of change with time. **2** Heat transfer can be approximated as being one-dimensional since it is predominantly in the *x*-direction. **3** Thermal conductivities are constant. **4** Heat transfer by radiation is negligible.

Properties The thermal conductivities are given to be $k = 0.72$ W/m · °C for bricks, $k = 0.22$ W/m · °C for plaster layers, and $k = 0.026$ W/m · °C for the rigid foam.

Analysis There is a pattern in the construction of this wall that repeats itself every 25-cm distance in the vertical direction. There is no variation in the horizontal direction. Therefore, we consider a 1-m-deep and 0.25-m-high portion of the wall, since it is representative of the entire wall.

Assuming any cross section of the wall normal to the *x*-direction to be *isothermal,* the thermal resistance network for the representative section of the wall becomes as shown in Fig. 3–21. The individual resistances are evaluated as:

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(10\ \text{W/m}^2 \cdot {}^\circ\text{C})(0.25 \times 1\ \text{m}^2)} = 0.4^\circ\text{C/W}$$

$$R_1 = R_{\text{foam}} = \frac{L}{kA} = \frac{0.03\ \text{m}}{(0.026\ \text{W/m} \cdot {}^\circ\text{C})(0.25 \times 1\ \text{m}^2)} = 4.6^\circ\text{C/W}$$

$$R_2 = R_6 = R_{\text{plaster, side}} = \frac{L}{kA} = \frac{0.02\ \text{m}}{(0.22\ \text{W/m} \cdot {}^\circ\text{C})(0.25 \times 1\ \text{m}^2)}$$

$$= 0.36^\circ\text{C/W}$$

$$R_3 = R_5 = R_{\text{plaster, center}} = \frac{L}{kA} = \frac{0.16\ \text{m}}{(0.22\ \text{W/m} \cdot {}^\circ\text{C})(0.015 \times 1\ \text{m}^2)}$$

$$= 48.48^\circ\text{C/W}$$

$$R_4 = R_{\text{brick}} = \frac{L}{kA} = \frac{0.16 \text{ m}}{(0.72 \text{ W/m} \cdot °\text{C})(0.22 \times 1 \text{ m}^2)} = 1.01°\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A} = \frac{1}{(25 \text{ W/m}^2 \cdot °\text{C})(0.25 \times 1 \text{ m}^2)} = 0.16°\text{C/W}$$

The three resistances R_3, R_4, and R_5 in the middle are parallel, and their equivalent resistance is determined from

$$\frac{1}{R_{\text{mid}}} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{48.48} + \frac{1}{1.01} + \frac{1}{48.48} = 1.03 \text{ W/°C}$$

which gives

$$R_{\text{mid}} = 0.97°\text{C/W}$$

Now all the resistances are in series, and the total resistance is

$$\begin{aligned} R_{\text{total}} &= R_i + R_1 + R_2 + R_{\text{mid}} + R_6 + R_o \\ &= 0.4 + 4.6 + 0.36 + 0.97 + 0.36 + 0.16 \\ &= 6.85°\text{C/W} \end{aligned}$$

Then the steady rate of heat transfer through the wall becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{[20 - (-10)]°\text{C}}{6.85°\text{C/W}} = 4.38 \text{ W} \qquad \text{(per 0.25 m}^2 \text{ surface area)}$$

or $4.38/0.25 = 17.5$ W per m^2 area. The total area of the wall is $A = 3 \text{ m} \times 5 \text{ m} = 15 \text{ m}^2$. Then the rate of heat transfer through the entire wall becomes

$$\dot{Q}_{\text{total}} = (17.5 \text{ W/m}^2)(15 \text{ m}^2) = \mathbf{263 \text{ W}}$$

Heat Conduction in Cylinder and Spheres

Consider a long cylindrical layer (such as a circular pipe) of inner radius r_1, outer radius r_2, length L, and average thermal conductivity k (Fig. 3–24). The two surfaces of the cylindrical layer are maintained at constant temperatures T_1 and T_2. There is no heat generation in the layer and the thermal conductivity is constant. For one-dimensional heat conduction through the cylindrical layer, we have $T(r)$. Then Fourier's law of heat conduction for heat transfer through the cylindrical layer can be expressed as

$$\dot{Q}_{\text{cond, cyl}} = -kA\frac{dT}{dr} \qquad \text{(W)} \tag{3-35}$$

where $A = 2\pi rL$ is the heat transfer area at location r. Note that A depends on r, and thus it *varies* in the direction of heat transfer. Separating the variables in the above equation and integrating from $r = r_1$, where $T(r_1) = T_1$, to $r = r_2$, where $T(r_2) = T_2$, gives

$$\int_{r=r_1}^{r_2} \frac{\dot{Q}_{\text{cond, cyl}}}{A}\, dr = -\int_{T=T_1}^{T_2} k\, dT \tag{3-36}$$

Substituting $A = 2\pi rL$ and performing the integrations give

$$\dot{Q}_{\text{cond, cyl}} = 2\pi Lk\frac{T_1 - T_2}{\ln(r_2/r_1)} \qquad \text{(W)} \tag{3-37}$$

since $\dot{Q}_{\text{cond, cyl}}$ = constant. This equation can be rearranged as

$$\dot{Q}_{\text{cond, cyl}} = \frac{T_1 - T_2}{R_{\text{cyl}}} \qquad \text{(W)} \tag{3-38}$$

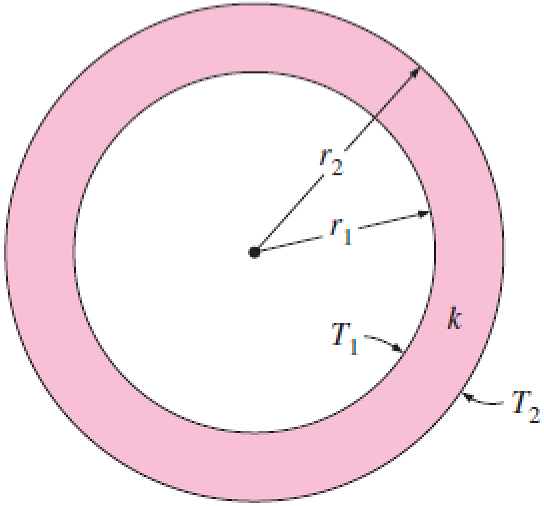


FIGURE 3–24
A long cylindrical pipe (or spherical shell) with specified inner and outer surface temperatures T_1 and T_2.

where

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi Lk} = \frac{\ln(\text{Outer radius/Inner radius})}{2\pi \times (\text{Length}) \times (\text{Thermal conductivity})} \quad \text{(3-39)}$$

is the *thermal resistance* of the cylindrical layer against heat conduction, or simply the **conduction resistance** of the cylinder layer.

We can repeat the analysis above for a *spherical layer* by taking $A = 4\pi r^2$ and performing the integrations in Eq. 3–36. The result can be expressed as

$$\dot{Q}_{cond,\ sph} = \frac{T_1 - T_2}{R_{sph}} \quad \text{(3-40)}$$

where

$$R_{sph} = \frac{r_2 - r_1}{4\pi r_1 r_2 k} = \frac{\text{Outer radius} - \text{Inner radius}}{4\pi(\text{Outer radius})(\text{Inner radius})(\text{Thermal conductivity})} \quad \text{(3-41)}$$

is the *thermal resistance* of the spherical layer against heat conduction, or simply the **conduction resistance** of the spherical layer.

Now consider steady one-dimensional heat flow through a cylindrical or spherical layer that is exposed to convection on both sides to fluids at temperatures $T_{\infty 1}$ and $T_{\infty 2}$ with heat transfer coefficients h_1 and h_2, respectively, as shown in Fig. 3–25. The thermal resistance network in this case consists of one conduction and two convection resistances in series, just like the one for the plane wall, and the rate of heat transfer under steady conditions can be expressed as

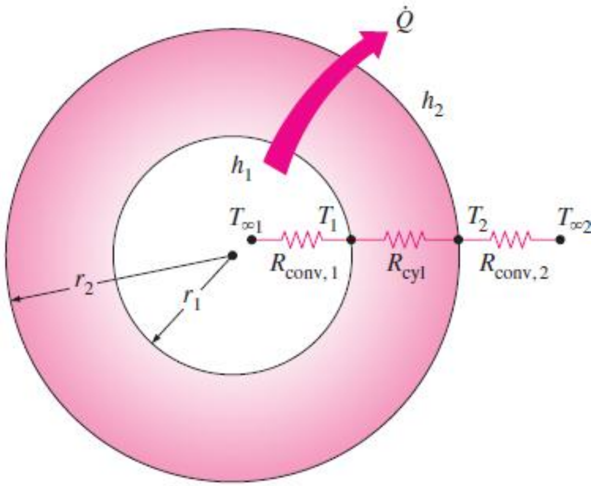
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} \tag{3-42}$$

where

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{cyl}} + R_{\text{conv, 2}} \\ &= \frac{1}{(2\pi r_1 L)h_1} + \frac{\ln(r_2/r_1)}{2\pi L k} + \frac{1}{(2\pi r_2 L)h_2} \end{aligned} \tag{3-43}$$

for a *cylindrical* layer, and

$$\begin{aligned} R_{\text{total}} &= R_{\text{conv, 1}} + R_{\text{sph}} + R_{\text{conv, 2}} \\ &= \frac{1}{(4\pi r_1^2)h_1} + \frac{r_2 - r_1}{4\pi r_1 r_2 k} + \frac{1}{(4\pi r_2^2)h_2} \end{aligned} \tag{3-44}$$



$R_{\text{total}} = R_{\text{conv},1} + R_{\text{cyl}} + R_{\text{conv},2}$

for a spherical layer. Note that A in the convection resistance relation $R_{\text{conv}} = 1/hA$ is the *surface area at which convection occurs.* It is equal to $A = 2\pi rL$ for a cylindrical surface and $A = 4\pi r^2$ for a spherical surface of radius r. Also note that the thermal resistances are in series, and thus the total thermal resistance is determined by simply adding the individual resistances, just like the electrical resistances connected in series.

Multilayer Cylinder and Spheres

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}}$$

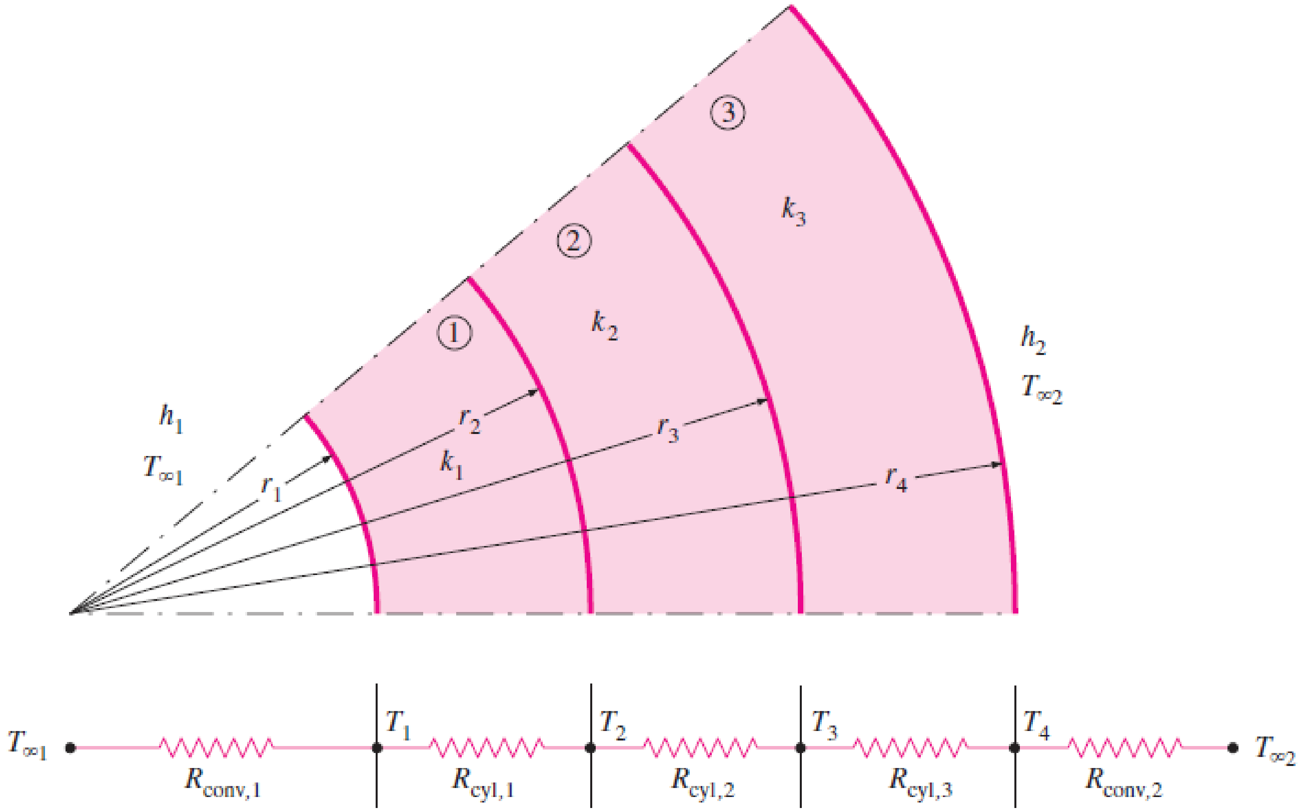


FIGURE 3–26
The thermal resistance network for heat transfer through a three-layered composite cylinder subjected to convection on both sides.

where R_{total} is the *total thermal resistance,* expressed as

$$R_{\text{total}} = R_{\text{conv, 1}} + R_{\text{cyl, 1}} + R_{\text{cyl, 2}} + R_{\text{cyl, 3}} + R_{\text{conv, 2}}$$

$$= \frac{1}{h_1 A_1} + \frac{\ln(r_2/r_1)}{2\pi L k_1} + \frac{\ln(r_3/r_2)}{2\pi L k_2} + \frac{\ln(r_4/r_3)}{2\pi L k_3} + \frac{1}{h_2 A_4}$$

EXAMPLE 3–8 **Heat Loss through an Insulated Steam Pipe**

Steam at $T_{\infty 1} = 320°C$ flows in a cast iron pipe ($k = 80$ W/m · °C) whose inner and outer diameters are $D_1 = 5$ cm and $D_2 = 5.5$ cm, respectively. The pipe is covered with 3-cm-thick glass wool insulation with $k = 0.05$ W/m · °C. Heat is lost to the surroundings at $T_{\infty 2} = 5°C$ by natural convection and radiation, with a combined heat transfer coefficient of $h_2 = 18$ W/m^2 · °C. Taking the heat transfer coefficient inside the pipe to be $h_1 = 60$ W/m^2 · °C, determine the rate of heat loss from the steam per unit length of the pipe. Also determine the temperature drops across the pipe shell and the insulation.

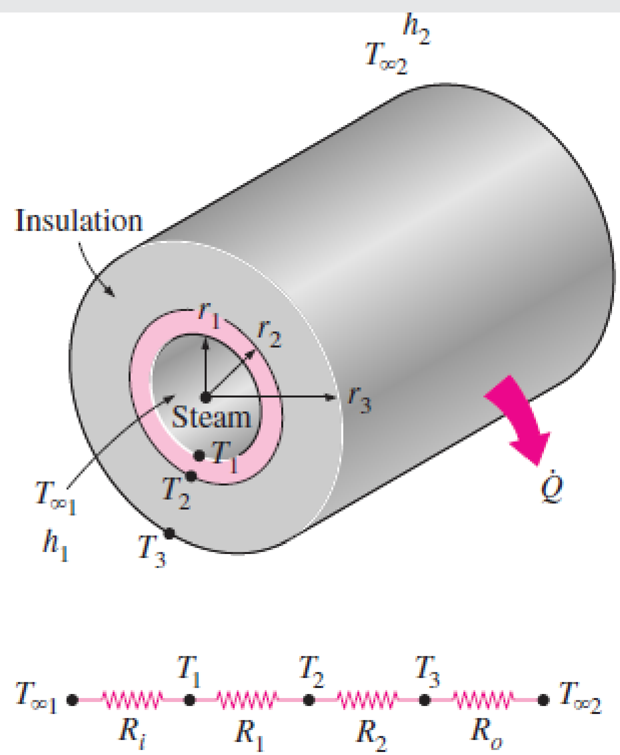


FIGURE 3–29
Schematic for Example 3–8.

SOLUTION A steam pipe covered with glass wool insulation is subjected to convection on its surfaces. The rate of heat transfer per unit length and the temperature drops across the pipe and the insulation are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible.

Properties The thermal conductivities are given to be $k = 80$ W/m · °C for cast iron and $k = 0.05$ W/m · °C for glass wool insulation.

Analysis The thermal resistance network for this problem involves four resistances in series and is given in Fig. 3–29. Taking $L = 1$ m, the areas of the surfaces exposed to convection are determined to be

$$A_1 = 2\pi r_1 L = 2\pi(0.025 \text{ m})(1 \text{ m}) = 0.157 \text{ m}^2$$
$$A_3 = 2\pi r_3 L = 2\pi(0.0575 \text{ m})(1 \text{ m}) = 0.361 \text{ m}^2$$

Then the individual thermal resistances become

$$R_i = R_{\text{conv, 1}} = \frac{1}{h_1 A} = \frac{1}{(60 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.157 \text{ m}^2)} = 0.106^\circ\text{C/W}$$

$$R_1 = R_{\text{pipe}} = \frac{\ln(r_2/r_1)}{2\pi k_1 L} = \frac{\ln(2.75/2.5)}{2\pi(80 \text{ W/m} \cdot {}^\circ\text{C})(1 \text{ m})} = 0.0002^\circ\text{C/W}$$

$$R_2 = R_{\text{insulation}} = \frac{\ln(r_3/r_2)}{2\pi k_2 L} = \frac{\ln(5.75/2.75)}{2\pi(0.05 \text{ W/m} \cdot {}^\circ\text{C})(1 \text{ m})} = 2.35^\circ\text{C/W}$$

$$R_o = R_{\text{conv, 2}} = \frac{1}{h_2 A_3} = \frac{1}{(18 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.361 \text{ m}^2)} = 0.154^\circ\text{C/W}$$

Noting that all resistances are in series, the total resistance is determined to be

$$R_{\text{total}} = R_i + R_1 + R_2 + R_o = 0.106 + 0.0002 + 2.35 + 0.154 = 2.61^\circ\text{C/W}$$

Then the steady rate of heat loss from the steam becomes

$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{total}}} = \frac{(320 - 5)^\circ\text{C}}{2.61^\circ\text{C/W}} = \mathbf{121 \text{ W}} \qquad \text{(per m pipe length)}$$

The heat loss for a given pipe length can be determined by multiplying the above quantity by the pipe length L.

The temperature drops across the pipe and the insulation are determined from Eq. 3–17 to be

$$\Delta T_{\text{pipe}} = \dot{Q} R_{\text{pipe}} = (121 \text{ W})(0.0002^\circ\text{C/W}) = \mathbf{0.02^\circ C}$$
$$\Delta T_{\text{insulation}} = \dot{Q} R_{\text{insulation}} = (121 \text{ W})(2.35^\circ\text{C/W}) = \mathbf{284^\circ C}$$

That is, the temperatures between the inner and the outer surfaces of the pipe differ by 0.02°C, whereas the temperatures between the inner and the outer surfaces of the insulation differ by 284°C.

Critical Radius of Insulation

- Adding more insulation to a wall or to the attic always decreases heat transfer.
- Adding insulation to a cylindrical pipe or a spherical shell, however, is a different matter.
- The additional insulation increases the conduction resistance of the insulation layer but decreases the convection resistance of the surface because of the increase in the outer surface area for convection.

Consider a cylindrical pipe of outer radius r_1 whose outer surface temperature T_1 is maintained constant (Fig. 3–30). The pipe is now insulated with a material whose thermal conductivity is k and outer radius is r_2. Heat is lost from the pipe to the surrounding medium at temperature T_∞, with a convection heat transfer coefficient h. The rate of heat transfer from the insulated pipe to the surrounding air can be expressed as (Fig. 3–31)

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{ins} + R_{conv}} = \frac{T_1 - T_\infty}{\dfrac{\ln(r_2/r_1)}{2\pi Lk} + \dfrac{1}{h(2\pi r_2 L)}} \qquad \textbf{(3-49)}$$

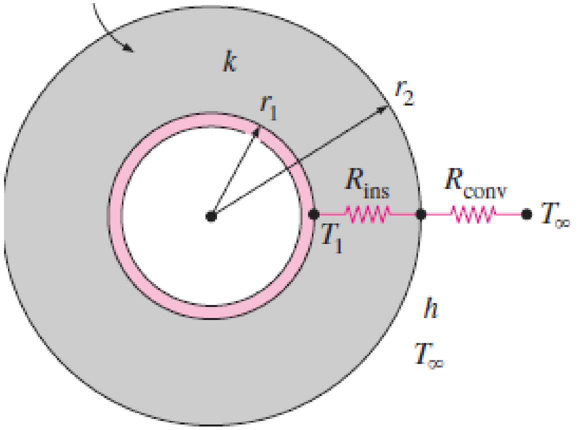


FIGURE 3–30
An insulated cylindrical pipe exposed to convection from the outer surface and the thermal resistance network associated with it.

The variation of $\dot{Q}$ with the outer radius of the insulation r_2 is plotted in Fig. 3–31. The value of r_2 at which $\dot{Q}$ reaches a maximum is determined from the requirement that $d\dot{Q}/dr_2 = 0$ (zero slope). Performing the differentiation and solving for r_2 yields the **critical radius of insulation** for a cylindrical body to be

$$r_{cr,\ cylinder} = \frac{k}{h} \qquad \text{(m)} \qquad \textbf{(3-50)}$$

The discussions above can be repeated for a sphere, and it can be shown in a similar manner that the critical radius of insulation for a spherical shell is

$$r_{cr,\ sphere} = \frac{2k}{h} \qquad \textbf{(3-51)}$$

where k is the thermal conductivity of the insulation and h is the convection heat transfer coefficient on the outer surface.

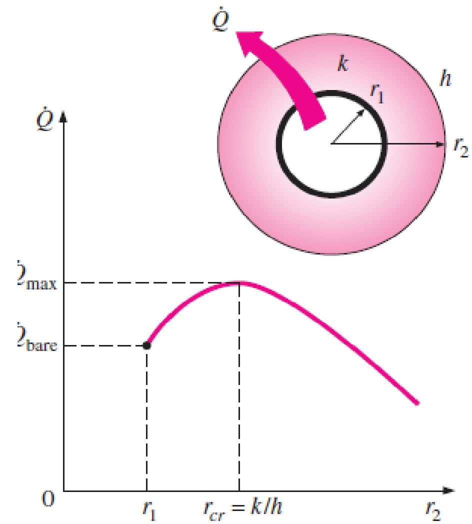


FIGURE 3–31

EXAMPLE 3–9 Heat Loss from an Insulated Electric Wire

A 3-mm-diameter and 5-m-long electric wire is tightly wrapped with a 2-mm-thick plastic cover whose thermal conductivity is $k = 0.15$ W/m · °C. Electrical measurements indicate that a current of 10 A passes through the wire and there is a voltage drop of 8 V along the wire. If the insulated wire is exposed to a medium at $T_\infty = 30°C$ with a heat transfer coefficient of $h = 12$ W/m² · °C, determine the temperature at the interface of the wire and the plastic cover in steady operation. Also determine whether doubling the thickness of the plastic cover will increase or decrease this interface temperature.

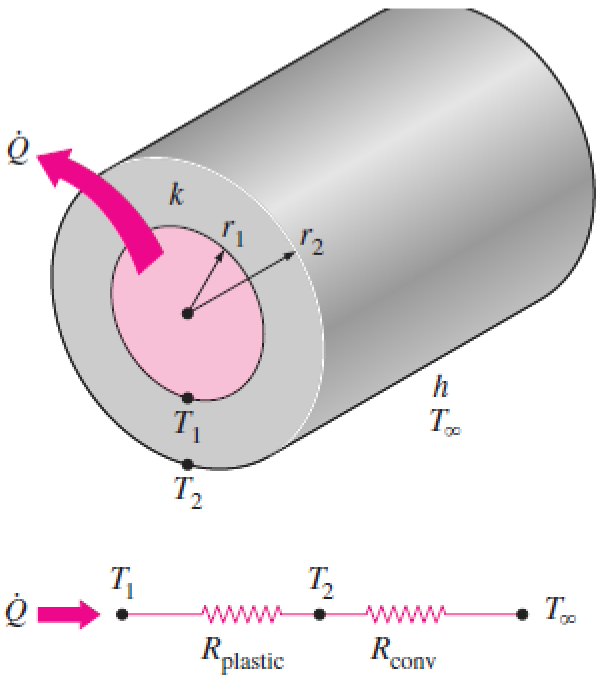


FIGURE 3–32
Schematic for Example 3–9.

Prepared by: Dr. Tahir Abdul Hussain

SOLUTION An electric wire is tightly wrapped with a plastic cover. The interface temperature and the effect of doubling the thickness of the plastic cover on the interface temperature are to be determined.

Assumptions **1** Heat transfer is steady since there is no indication of any change with time. **2** Heat transfer is one-dimensional since there is thermal symmetry about the centerline and no variation in the axial direction. **3** Thermal conductivities are constant. **4** The thermal contact resistance at the interface is negligible. **5** Heat transfer coefficient incorporates the radiation effects, if any.

Properties The thermal conductivity of plastic is given to be $k = 0.15$ W/m · °C.

Analysis Heat is generated in the wire and its temperature rises as a result of resistance heating. We assume heat is generated uniformly throughout the wire and is transferred to the surrounding medium in the radial direction. In steady operation, the rate of heat transfer becomes equal to the heat generated within the wire, which is determined to be

$$\dot{Q} = \dot{W}_e = VI = (8 \text{ V})(10 \text{ A}) = 80 \text{ W}$$

The thermal resistance network for this problem involves a conduction resistance for the plastic cover and a convection resistance for the outer surface in series, as shown in Fig. 3–32. The values of these two resistances are determined to be

$$A_2 = (2\pi r_2)L = 2\pi(0.0035 \text{ m})(5 \text{ m}) = 0.110 \text{ m}^2$$

$$R_{\text{conv}} = \frac{1}{hA_2} = \frac{1}{(12 \text{ W/m}^2 \cdot {}^\circ\text{C})(0.110 \text{ m}^2)} = 0.76^\circ\text{C/W}$$

$$R_{\text{plastic}} = \frac{\ln(r_2/r_1)}{2\pi kL} = \frac{\ln(3.5/1.5)}{2\pi(0.15 \text{ W/m} \cdot {}^\circ\text{C})(5 \text{ m})} = 0.18^\circ\text{C/W}$$

and therefore

$$R_{\text{total}} = R_{\text{plastic}} + R_{\text{conv}} = 0.76 + 0.18 = 0.94°\text{C/W}$$

Then the interface temperature can be determined from

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{total}}} \longrightarrow T_1 = T_\infty + \dot{Q}R_{\text{total}}$$

$$= 30°\text{C} + (80\text{ W})(0.94°\text{C/W}) = \mathbf{105°C}$$

Note that we did not involve the electrical wire directly in the thermal resistance network, since the wire involves heat generation.

To answer the second part of the question, we need to know the critical radius of insulation of the plastic cover. It is determined from Eq. 3–50 to be

$$r_{cr} = \frac{k}{h} = \frac{0.15\text{ W/m}\cdot°\text{C}}{12\text{ W/m}^2\cdot°\text{C}} = 0.0125\text{ m} = 12.5\text{ mm}$$

which is larger than the radius of the plastic cover. Therefore, increasing the thickness of the plastic cover will *enhance* heat transfer until the outer radius of the cover reaches 12.5 mm. As a result, the rate of heat transfer $\dot{Q}$ will *increase* when the interface temperature T_1 is held constant, or T_1 will *decrease* when $\dot{Q}$ is held constant, which is the case here.

Heat Transfer from Finned Surface

Consider a volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and a perimeter of p, as shown in Fig. 3–35. Under steady conditions, the energy balance on this volume element can be expressed as

$$\begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ into} \\ \text{the element at } x\end{pmatrix} = \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{conduction} \text{ from the} \\ \text{element at } x + \Delta x\end{pmatrix} + \begin{pmatrix}\text{Rate of } \textit{heat} \\ \textit{convection} \text{ from} \\ \text{the element}\end{pmatrix}$$

or

$$\dot{Q}_{\text{cond}, x} = \dot{Q}_{\text{cond}, x+\Delta x} + \dot{Q}_{\text{conv}}$$

where

$$\dot{Q}_{\text{conv}} = h(p\,\Delta x)(T - T_\infty)$$

Substituting and dividing by Δx, we obtain

$$\frac{\dot{Q}_{\text{cond}, x+\Delta x} - \dot{Q}_{\text{cond}, x}}{\Delta x} + hp(T - T_\infty) = 0 \qquad \textbf{(3-52)}$$

Taking the limit as $\Delta x \to 0$ gives

$$\frac{d\dot{Q}_{\text{cond}}}{dx} + hp(T - T_\infty) = 0 \qquad \textbf{(3-53)}$$

From Fourier's law of heat conduction we have

$$\dot{Q}_{\text{cond}} = -kA_c\frac{dT}{dx} \qquad \textbf{(3-54)}$$

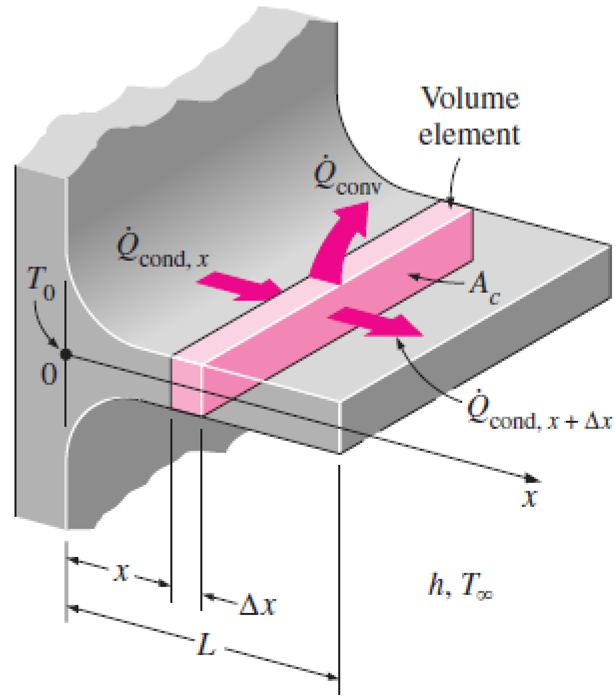


FIGURE 3–35
Volume element of a fin at location x having a length of Δx, cross-sectional area of A_c, and perimeter of p.

where A_c is the cross-sectional area of the fin at location x. Substitution of this relation into Eq. 3–53 gives the differential equation governing heat transfer in fins,

$$\frac{d}{dx}\left(kA_c\frac{dT}{dx}\right) - hp(T - T_\infty) = 0 \tag{3-55}$$

In general, the cross-sectional area A_c and the perimeter p of a fin vary with x, which makes this differential equation difficult to solve. In the special case of *constant cross section* and *constant thermal conductivity,* the differential equation 3–55 reduces to

$$\frac{d^2\theta}{dx^2} - a^2\theta = 0 \tag{3-56}$$

where

$$a^2 = \frac{hp}{kA_c} \tag{3-57}$$

and $\theta = T - T_\infty$ is the *temperature excess.* At the fin base we have $\theta_b = T_b - T_\infty$.

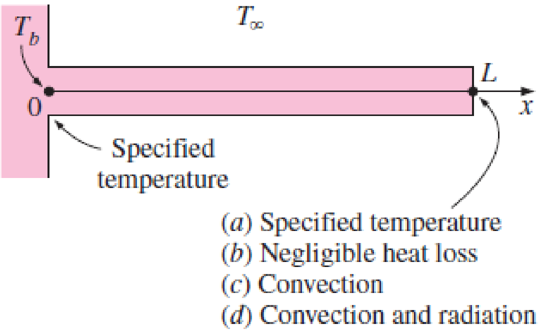


FIGURE 3–36
Boundary conditions at the fin base and the fin tip.

$$\theta(x) = C_1e^{ax} + C_2e^{-ax} \tag{3-58}$$

Boundary condition at fin base: $\quad \theta(0) = \theta_b = T_b - T_\infty \quad$ **(3-59)**

Infinitely Long Pipe

Boundary condition at fin tip: $\theta(L) = T(L) - T_\infty = 0 \quad \text{as} \quad L \to \infty$

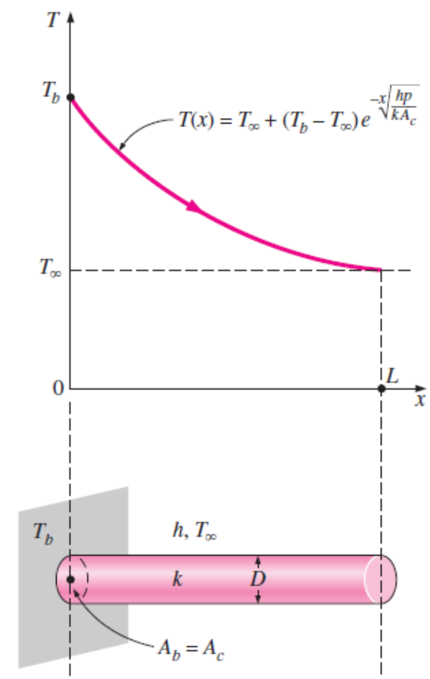
Very long fin:
$$\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-ax} = e^{-x\sqrt{hp/kA_c}} \qquad \text{(3-60)}$$

Note that the temperature along the fin in this case decreases *exponentially* from T_b to T_∞, as shown in Fig. 3–37. The steady rate of *heat transfer* from the entire fin can be determined from Fourier's law of heat conduction

Very long fin:
$$\dot{Q}_{\text{long fin}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0} = \sqrt{hpkA_c}\,(T_b - T_\infty) \qquad \text{(3-61)}$$

where p is the perimeter, A_c is the cross-sectional area of the fin, and x is the distance from the fin base. Alternatively, the rate of heat transfer from the fin could also be determined by considering heat transfer from a differential volume element of the fin and integrating it over the entire surface of the fin. That is,

$$\dot{Q}_{\text{fin}} = \int_{A_{\text{fin}}} h[T(x) - T_\infty]\, dA_{\text{fin}} = \int_{A_{\text{fin}}} h\theta(x)\, dA_{\text{fin}} \qquad \text{(3-62)}$$



($p = \pi D$, $A_c = \pi D^2/4$ for a cylindrical fin)

FIGURE 3–37
A long circular fin of uniform cross section and the variation of temperature along it.

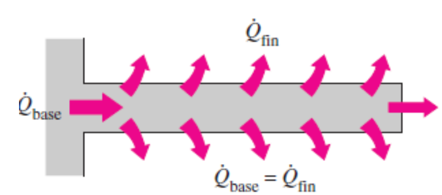


FIGURE 3–38
Under steady conditions, heat transfer from the exposed surfaces of the fin is equal to heat conduction to the fin at the base.

Negligible Heat Loss from Fin Tip (Insulated Fin Tip)

Boundary condition at fin tip:

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0 \qquad \textbf{(3-63)}$$

Adiabatic fin tip:

$$\frac{T(x) - T_\infty}{T_b - T_\infty} = \frac{\cosh a(L - x)}{\cosh aL} \qquad \textbf{(3-64)}$$

The rate of heat transfer from the fin can be determined again from Fourier's law of heat conduction:

Adiabatic fin tip:

$$\dot{Q}_{\text{insulated tip}} = -kA_c \left.\frac{dT}{dx}\right|_{x=0}$$

$$= \sqrt{hpkA_c}\,(T_b - T_\infty)\tanh aL \qquad \textbf{(3-65)}$$

Note that the heat transfer relations for the very long fin and the fin with negligible heat loss at the tip differ by the factor tanh aL, which approaches 1 as L becomes very large.

Convection or Combined from Fin Tip

A practical way of accounting for the heat loss from the fin tip is to replace the *fin length L* in the relation for the *insulated tip* case by a **corrected length** defined as (Fig. 3–39)

Corrected fin length:

$$L_c = L + \frac{A_c}{p} \quad \textbf{(3-66)}$$

where A_c is the cross-sectional area and p is the perimeter of the fin at the tip. Multiplying the relation above by the perimeter gives $A_{\text{corrected}} = A_{\text{fin (lateral)}} + A_{\text{tip}}$, which indicates that the fin area determined using the corrected length is equivalent to the sum of the lateral fin area plus the fin tip area.

Using the proper relations for A_c and p, the corrected lengths for rectangular and cylindrical fins are easily determined to be

$$L_{c,\text{ rectangular fin}} = L + \frac{t}{2} \quad \text{and} \quad L_{c,\text{ cylindrical fin}} = L + \frac{D}{4}$$

where t is the thickness of the rectangular fins and D is the diameter of the cylindrical fins.

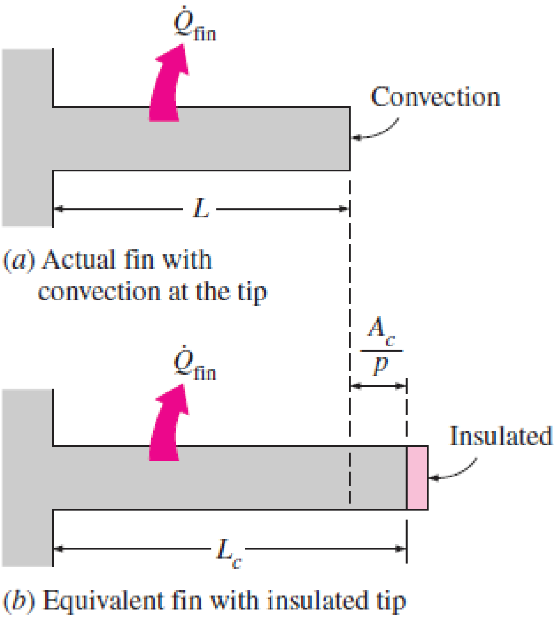


FIGURE 3–39
Corrected fin length L_c is defined such that heat transfer from a fin of length L_c with insulated tip is equal to heat transfer from the actual fin of length L with convection at the fin tip.